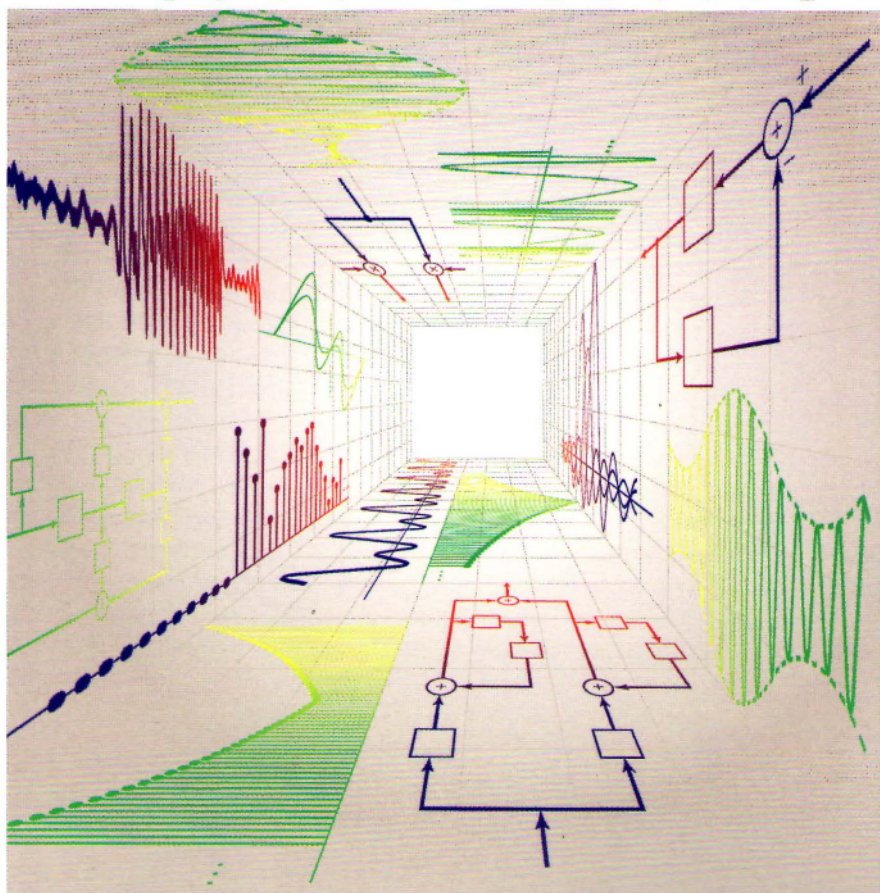


INTERNATIONAL EDITION

SIGNALS & SYSTEMS

SECOND EDITION



ALAN V. OPPENHEIM
ALAN S. WILLSKY
WITH S. HAMID NAWAB

PRENTICE HALL SIGNAL PROCESSING SERIES
ALAN V. OPPENHEIM, SERIES EDITOR



SECOND EDITION

SIGNALS & SYSTEMS

ALAN V. OPPENHEIM

ALAN S. WILLSKY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

WITH

S. HAMID NAWAB

BOSTON UNIVERSITY



PRENTICE-HALL INTERNATIONAL, INC.

CONTENTS

PREFACE xvii

ACKNOWLEDGMENTS xxv

FOREWORD xxvii

1 SIGNALS AND SYSTEMS 1

1.0 Introduction 1

1.1 Continuous-Time and Discrete-Time Signals 1

1.1.1 Examples and Mathematical Representation 1

1.1.2 Signal Energy and Power 5

1.2 Transformations of the Independent Variable 7

1.2.1 Examples of Transformations of the Independent Variable 8

1.2.2 Periodic Signals 11

1.2.3 Even and Odd Signals 13

1.3 Exponential and Sinusoidal Signals 14

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signals 15

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals 21

1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials 25

1.4 The Unit Impulse and Unit Step Functions 30

1.4.1 The Discrete-Time Unit Impulse and Unit Step Sequences 30

1.4.2 The Continuous-Time Unit Step and Unit Impulse Functions 32

1.5 Continuous-Time and Discrete-Time Systems 38

1.5.1 Simple Examples of Systems 39

1.5.2 Interconnections of Systems 41

1.6 Basic System Properties 44

1.6.1 Systems with and without Memory 44

1.6.2 Invertibility and Inverse Systems 45

1.6.3 Causality 46

1.6.4 Stability 48

1.6.5 Time Invariance 50

1.6.6 Linearity 53

1.7 Summary 56

Problems 57

2 LINEAR TIME-INVARIANT SYSTEMS 74

2.0 Introduction 74

2.1 Discrete-Time LTI Systems: The Convolution Sum 75

- 2.1.1 The Representation of Discrete-Time Signals in Terms of Impulses 75
- 2.1.2 The Discrete-Time Unit Impulse Response and the Convolution-Sum Representation of LTI Systems 77
- 2.2 Continuous-Time LTI Systems: The Convolution Integral 90**
 - 2.2.1 The Representation of Continuous-Time Signals in Terms of Impulses 90
 - 2.2.2 The Continuous-Time Unit Impulse Response and the Convolution Integral Representation of LTI Systems 94
- 2.3 Properties of Linear Time-Invariant Systems 103**
 - 2.3.1 The Commutative Property 104
 - 2.3.2 The Distributive Property 104
 - 2.3.3 The Associative Property 107
 - 2.3.4 LTI Systems with and without Memory 108
 - 2.3.5 Invertibility of LTI Systems 109
 - 2.3.6 Causality for LTI Systems 112
 - 2.3.7 Stability for LTI Systems 113
 - 2.3.8 The Unit Step Response of an LTI System 115
- 2.4 Causal LTI Systems Described by Differential and Difference Equations 116**
 - 2.4.1 Linear Constant-Coefficient Differential Equations 117
 - 2.4.2 Linear Constant-Coefficient Difference Equations 121
 - 2.4.3 Block Diagram Representations of First-Order Systems Described by Differential and Difference Equations 124
- 2.5 Singularity Functions 127**
 - 2.5.1 The Unit Impulse as an Idealized Short Pulse 128
 - 2.5.2 Defining the Unit Impulse through Convolution 131
 - 2.5.3 Unit Doublets and Other Singularity Functions 132
- 2.6 Summary 137**
 - Problems 137**

3 FOURIER SERIES REPRESENTATION OF PERIODIC SIGNALS 177

- 3.0 Introduction 177**
- 3.1 A Historical Perspective 178**
- 3.2 The Response of LTI Systems to Complex Exponentials 182**
- 3.3 Fourier Series Representation of Continuous-Time Periodic Signals 186**
 - 3.3.1 Linear Combinations of Harmonically Related Complex Exponentials 186
 - 3.3.2 Determination of the Fourier Series Representation of a Continuous-Time Periodic Signal 190
- 3.4 Convergence of the Fourier Series 195**

- 3.5 Properties of Continuous-Time Fourier Series 202**
 - 3.5.1 Linearity 202
 - 3.5.2 Time Shifting 202
 - 3.5.3 Time Reversal 203
 - 3.5.4 Time Scaling 204
 - 3.5.5 Multiplication 204
 - 3.5.6 Conjugation and Conjugate Symmetry 204
 - 3.5.7 Parseval's Relation for Continuous-Time Periodic Signals 205
 - 3.5.8 Summary of Properties of the Continuous-Time Fourier Series 205
 - 3.5.9 Examples 205
- 3.6 Fourier Series Representation of Discrete-Time Periodic Signals 211**
 - 3.6.1 Linear Combinations of Harmonically Related Complex Exponentials 211
 - 3.6.2 Determination of the Fourier Series Representation of a Periodic Signal 212
- 3.7 Properties of Discrete-Time Fourier Series 221**
 - 3.7.1 Multiplication 222
 - 3.7.2 First Difference 222
 - 3.7.3 Parseval's Relation for Discrete-Time Periodic Signals 223
 - 3.7.4 Examples 223
- 3.8 Fourier Series and LTI Systems 226**
- 3.9 Filtering 231**
 - 3.9.1 Frequency-Shaping Filters 232
 - 3.9.2 Frequency-Selective Filters 236
- 3.10 Examples of Continuous-Time Filters Described by Differential Equations 239**
 - 3.10.1 A Simple *RC* Lowpass Filter 239
 - 3.10.2 A Simple *RC* Highpass Filter 241
- 3.11 Examples of Discrete-Time Filters Described by Difference Equations 244**
 - 3.11.1 First-Order Recursive Discrete-Time Filters 244
 - 3.11.2 Nonrecursive Discrete-Time Filters 245
- 3.12 Summary 249**
 - Problems 250

4 THE CONTINUOUS-TIME FOURIER TRANSFORM 284

- 4.0 Introduction 284**
- 4.1 Representation of Aperiodic Signals: The Continuous-Time Fourier Transform 285**
 - 4.1.1 Development of the Fourier Transform Representation of an Aperiodic Signal 285
 - 4.1.2 Convergence of Fourier Transforms 289
 - 4.1.3 Examples of Continuous-Time Fourier Transforms 290

- 4.2 The Fourier Transform for Periodic Signals 296**
- 4.3 Properties of the Continuous-Time Fourier Transform 300**
 - 4.3.1 Linearity 301
 - 4.3.2 Time Shifting 301
 - 4.3.3 Conjugation and Conjugate Symmetry 303
 - 4.3.4 Differentiation and Integration 306
 - 4.3.5 Time and Frequency Scaling 308
 - 4.3.6 Duality 309
 - 4.3.7 Parseval's Relation 312
- 4.4 The Convolution Property 314**
 - 4.4.1 Examples 317
- 4.5 The Multiplication Property 322**
 - 4.5.1 Frequency-Selective Filtering with Variable Center Frequency 325
- 4.6 Tables of Fourier Properties and of Basic Fourier Transform Pairs 328**
- 4.7 Systems Characterized by Linear Constant-Coefficient Differential Equations 330**
- 4.8 Summary 333**
 - Problems 334

5 THE DISCRETE-TIME FOURIER TRANSFORM 358

- 5.0 Introduction 358**
- 5.1 Representation of Aperiodic Signals: The Discrete-Time Fourier Transform 359**
 - 5.1.1 Development of the Discrete-Time Fourier Transform 359
 - 5.1.2 Examples of Discrete-Time Fourier Transforms 362
 - 5.1.3 Convergence Issues Associated with the Discrete-Time Fourier Transform 366
- 5.2 The Fourier Transform for Periodic Signals 367**
- 5.3 Properties of the Discrete-Time Fourier Transform 372**
 - 5.3.1 Periodicity of the Discrete-Time Fourier Transform 373
 - 5.3.2 Linearity of the Fourier Transform 373
 - 5.3.3 Time Shifting and Frequency Shifting 373
 - 5.3.4 Conjugation and Conjugate Symmetry 375
 - 5.3.5 Differencing and Accumulation 375
 - 5.3.6 Time Reversal 376
 - 5.3.7 Time Expansion 377
 - 5.3.8 Differentiation in Frequency 380
 - 5.3.9 Parseval's Relation 380
- 5.4 The Convolution Property 382**
 - 5.4.1 Examples 383
- 5.5 The Multiplication Property 388**
- 5.6 Tables of Fourier Transform Properties and Basic Fourier Transform Pairs 390**

- 5.7 Duality 390**
 - 5.7.1 Duality in the Discrete-Time Fourier Series 391
 - 5.7.2 Duality between the Discrete-Time Fourier Transform and the Continuous-Time Fourier Series 395
- 5.8 Systems Characterized by Linear Constant-Coefficient Difference Equations 396**
- 5.9 Summary 399**
 - Problems 400

6 TIME AND FREQUENCY CHARACTERIZATION OF SIGNALS AND SYSTEMS 423

- 6.0 Introduction 423**
- 6.1 The Magnitude-Phase Representation of the Fourier Transform 423**
- 6.2 The Magnitude-Phase Representation of the Frequency Response of LTI Systems 427**
 - 6.2.1 Linear and Nonlinear Phase 428
 - 6.2.2 Group Delay 430
 - 6.2.3 Log-Magnitude and Bode Plots 436
- 6.3 Time-Domain Properties of Ideal Frequency-Selective Filters 439**
- 6.4 Time-Domain and Frequency-Domain Aspects of Nonideal Filters 444**
- 6.5 First-Order and Second-Order Continuous-Time Systems 448**
 - 6.5.1 First-Order Continuous-Time Systems 448
 - 6.5.2 Second-Order Continuous-Time Systems 451
 - 6.5.3 Bode Plots for Rational Frequency Responses 456
- 6.6 First-Order and Second-Order Discrete-Time Systems 461**
 - 6.6.1 First-Order Discrete-Time Systems 461
 - 6.6.2 Second-Order Discrete-Time Systems 465
- 6.7 Examples of Time- and Frequency-Domain Analysis of Systems 472**
 - 6.7.1 Analysis of an Automobile Suspension System 473
 - 6.7.2 Examples of Discrete-Time Nonrecursive Filters 476
- 6.8 Summary 482**
 - Problems 483

7 SAMPLING 514

- 7.0 Introduction 514**
- 7.1 Representation of a Continuous-Time Signal by Its Samples: The Sampling Theorem 515**
 - 7.1.1 Impulse-Train Sampling 516
 - 7.1.2 Sampling with a Zero-Order Hold 520

- 7.2 Reconstruction of a Signal from Its Samples Using Interpolation 522**
- 7.3 The Effect of Undersampling: Aliasing 527**
- 7.4 Discrete-Time Processing of Continuous-Time Signals 534**
 - 7.4.1 Digital Differentiator 541
 - 7.4.2 Half-Sample Delay 543
- 7.5 Sampling of Discrete-Time Signals 545**
 - 7.5.1 Impulse-Train Sampling 545
 - 7.5.2 Discrete-Time Decimation and Interpolation 549
- 7.6 Summary 555**
- Problems 556**

8 COMMUNICATION SYSTEMS 582

- 8.0 Introduction 582**
- 8.1 Complex Exponential and Sinusoidal Amplitude Modulation 583**
 - 8.1.1 Amplitude Modulation with a Complex Exponential Carrier 583
 - 8.1.2 Amplitude Modulation with a Sinusoidal Carrier 585
- 8.2 Demodulation for Sinusoidal AM 587**
 - 8.2.1 Synchronous Demodulation 587
 - 8.2.2 Asynchronous Demodulation 590
- 8.3 Frequency-Division Multiplexing 594**
- 8.4 Single-Sideband Sinusoidal Amplitude Modulation 597**
- 8.5 Amplitude Modulation with a Pulse-Train Carrier 601**
 - 8.5.1 Modulation of a Pulse-Train Carrier 601
 - 8.5.2 Time-Division Multiplexing 604
- 8.6 Pulse-Amplitude Modulation 604**
 - 8.6.1 Pulse-Amplitude Modulated Signals 604
 - 8.6.2 Intersymbol Interference in PAM Systems 607
 - 8.6.3 Digital Pulse-Amplitude and Pulse-Code Modulation 610
- 8.7 Sinusoidal Frequency Modulation 611**
 - 8.7.1 Narrowband Frequency Modulation 613
 - 8.7.2 Wideband Frequency Modulation 615
 - 8.7.3 Periodic Square-Wave Modulating Signal 617
- 8.8 Discrete-Time Modulation 619**
 - 8.8.1 Discrete-Time Sinusoidal Amplitude Modulation 619
 - 8.8.2 Discrete-Time Transmodulation 623
- 8.9 Summary 623**
- Problems 625**

9 THE LAPLACE TRANSFORM 654

- 9.0 Introduction 654**
- 9.1 The Laplace Transform 655**
- 9.2 The Region of Convergence for Laplace Transforms 662**

- 9.3 The Inverse Laplace Transform 670**
- 9.4 Geometric Evaluation of the Fourier Transform from the Pole-Zero Plot 674**
 - 9.4.1 First-Order Systems 676
 - 9.4.2 Second-Order Systems 677
 - 9.4.3 All-Pass Systems 681
- 9.5 Properties of the Laplace Transform 682**
 - 9.5.1 Linearity of the Laplace Transform 683
 - 9.5.2 Time Shifting 684
 - 9.5.3 Shifting in the s -Domain 685
 - 9.5.4 Time Scaling 685
 - 9.5.5 Conjugation 687
 - 9.5.6 Convolution Property 687
 - 9.5.7 Differentiation in the Time Domain 688
 - 9.5.8 Differentiation in the s -Domain 688
 - 9.5.9 Integration in the Time Domain 690
 - 9.5.10 The Initial- and Final-Value Theorems 690
 - 9.5.11 Table of Properties 691
- 9.6 Some Laplace Transform Pairs 692**
- 9.7 Analysis and Characterization of LTI Systems Using the Laplace Transform 693**
 - 9.7.1 Causality 693
 - 9.7.2 Stability 695
 - 9.7.3 LTI Systems Characterized by Linear Constant-Coefficient Differential Equations 698
 - 9.7.4 Examples Relating System Behavior to the System Function 701
 - 9.7.5 Butterworth Filters 703
- 9.8 System Function Algebra and Block Diagram Representations 706**
 - 9.8.1 System Functions for Interconnections of LTI Systems 707
 - 9.8.2 Block Diagram Representations for Causal LTI Systems Described by Differential Equations and Rational System Functions 708
- 9.9 The Unilateral Laplace Transform 714**
 - 9.9.1 Examples of Unilateral Laplace Transforms 714
 - 9.9.2 Properties of the Unilateral Laplace Transform 716
 - 9.9.3 Solving Differential Equations Using the Unilateral Laplace Transform 719
- 9.10 Summary 720**
 - Problems 721

10 THE Z-TRANSFORM 741

- 10.0 Introduction 741**
- 10.1 The z -Transform 741**
- 10.2 The Region of Convergence for the z -Transform 748**

- 10.3 The Inverse z -Transform 757**
- 10.4 Geometric Evaluation of the Fourier Transform from the Pole-Zero Plot 763**
 - 10.4.1 First-Order Systems 763
 - 10.4.2 Second-Order Systems 765
- 10.5 Properties of the z -Transform 767**
 - 10.5.1 Linearity 767
 - 10.5.2 Time Shifting 767
 - 10.5.3 Scaling in the z -Domain 768
 - 10.5.4 Time Reversal 769
 - 10.5.5 Time Expansion 769
 - 10.5.6 Conjugation 770
 - 10.5.7 The Convolution Property 770
 - 10.5.8 Differentiation in the z -Domain 772
 - 10.5.9 The Initial-Value Theorem 773
 - 10.5.10 Summary of Properties 774
- 10.6 Some Common z -Transform Pairs 774**
- 10.7 Analysis and Characterization of LTI Systems Using z -Transforms 774**
 - 10.7.1 Causality 776
 - 10.7.2 Stability 777
 - 10.7.3 LTI Systems Characterized by Linear Constant-Coefficient Difference Equations 779
 - 10.7.4 Examples Relating System Behavior to the System Function 781
- 10.8 System Function Algebra and Block Diagram Representations 783**
 - 10.8.1 System Functions for Interconnections of LTI Systems 784
 - 10.8.2 Block Diagram Representations for Causal LTI Systems Described by Difference Equations and Rational System Functions 784
- 10.9 The Unilateral z -Transform 789**
 - 10.9.1 Examples of Unilateral z -Transforms and Inverse Transforms 790
 - 10.9.2 Properties of the Unilateral z -Transform 792
 - 10.9.3 Solving Difference Equations Using the Unilateral z -Transform 795
- 10.10 Summary 796**
 - Problems 797**

11 LINEAR FEEDBACK SYSTEMS 816

- 11.0 Introduction 816**
- 11.1 Linear Feedback Systems 819**
- 11.2 Some Applications and Consequences of Feedback 820**
 - 11.2.1 Inverse System Design 820
 - 11.2.2 Compensation for Nonideal Elements 821
 - 11.2.3 Stabilization of Unstable Systems 823

